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AUTHOR Bernard, John E.
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ABSTRACT

This paper describes the development of curriculum materials for teaching the Sum of Squared Errors (SSE) to one class of 25 eighth graders in Hawaii. Microcomputers were used in class. Prior to explicit introduction of the SSE students were given repeated contact with a data base of various statistics collected from members of their class. Then a computer program was introduced which randomly selected values using the data base; students tried different guessing strategies. Worksheets were used in connection with this and other content; they are included in the paper. Interactive game-like situations were also used. The bridge to standard deviation was then described. Misconceptions that students may bring to the discussion of probability are described, with ways the developed materials sought to clarify the concepts. Finally, the application of SSE to linear regression is discussed. (MNS)

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Teaching SSE and Some of Its Applications in Junior High School

by

John E. Bernard, Ph.D.

Department of Secondary Education
West Georgia College
Carrollton, Georgia 30118

This presentation is about the development of curriculum materials for teaching the SUM OF SQUARED ERRORS and some of its applications to eighth graders at the University of Hawaii Laboratory School. The work was done in a bigger context, that of implementing recommendations from major professional organizations and commissions for incorporating probability and statistics into the secondary school curriculum and for taking full advantage of the capabilities of microcomputers for improving the teaching of mathematics. (See the NACOME, NCTM Agenda for Action, and ICOTS-I reports, for example.)

The work was done with one class of 25 eighth graders during the spring semester of the 1984-85 school year. Class membership was balanced ethnically and socio-economically with 13 girls and 12 boys. Abilities ranged from stanine 3 to 9 on various scales of standardized tests. (See Table 1.)

Table 1

Distribution of Stanines for Selected Scales

Stanines	4	5	6	7	8	9	Means
Fall 1984							
MathConcepts		2	2	15	2	4	7.16
Computation		3	4	5	10	3	7.24
ProbSlvg		6	5	8	2	4	6.72
Fall 1985							
Quantitative	1	3	3	7	7	3	6.76

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Note: This paper was prepared for presentation Thursday August 14, 1986 (Cornett A121, 9:00am) at the Second International Conference for the Teaching of Statistics (ICOTS II), University of Victoria (University Extension Office, P.O. Box 1700), Victoria, British Columbia, Canada (V8W 2Y2).

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Project materials were taught in a separate, reoccurring strand along side of a pre-algebra curriculum that had been developed at the Lab School a few years earlier. That curriculum included such things as:

- Graphing
- Computations with signed numbers
- Work with algebraic expressions, equations, and formulas
- Collecting, organizing, and interpreting data (probabilistic and otherwise)
- Determining means, medians, and modes, and
- An intuitive approach to correlation.

Thus, the Lab School eighth grade mathematics curriculum most likely had more probability and statistics in it than most standard curriculums. Too, this material was actually taught; it was not just in a part of the text book that one just skipped.)

Computer facilities were essential. They included:

- A portable Apple IIe with a large 25" monitor and a smaller 12" monitor
- A microcomputer lab with about 17 Apples (enough to run 2 students to a unit), and
- An Apple IIe with a printer for office use.

When the developmental work was fully in progress, it touched upon as many as three class periods during a one-week span, especially if students were given a hands-on laboratory session to interact with the computer. In such cases, it was typical to use some or part of Thursday classes to introduce concepts, computer programs, and areas of inquiry. This was usually done with teacher-lead class disscussions and corresponding worksheets. Discussions concentrated on setting up lesson objectives and providing procedural information. Labs were then on Friday with the latter portion of such being used for discussion and/or to give students any additional clarification or directions that might be needed for take-home assignments. (Sometimes lab sessions were data generation and collection events which provided data that served as a basis for subsequent work.) An additional bit of time might have been used the following Monday when the students handed in their worksheets and/or, depending upon the situation, again when the worksheets were returned to the class. Other times there was less student contact with project materials. Thus, for the semester, student contact time with computer related probability and statistics lessons average about one to at most one and one-half class sessions per week.

Prior to an explicit introduction of the SSE, students were given repeated contact with a data base made up of various statistics collected from members of their class. Height, in inches, was included. Students had made tables and histograms, and had determined means, medians, and modes using the data base. Then a computer program was introduced which randomly selected values from the data base. Specifically, it was used to repeatedly display a single student's name and height, simultaneously, on the screen during a class discussion. Students were invited to try different guessing strategies and means for determining who, if anyone, was best at guessing the randomly selected heights. (Names were important, but only motivationally). "Being close" and "Doing it over the long-run" were reinforced by way of explicit teacher-lead discussion when students proposed such ideas. (No technical terminology was required nor interjected).

The following worksheet [Developing Good Guessing Strategies for Measurements, 2/7/85] was introduced and used in connection with the aforementioned computer program.

Dr. Bernard
2/7/85

Math Eight
Developing Good Guessing
Strategies for Measurements

Name_____

Date_____

You may either formulate a guessing strategy ahead of time, during, or after this activity. You may even want to check several strategies to see how they work and which, if any, might be the BEST. These are the reasons for this activity.

The computer will be used to randomly pick students from the class. Beforehand, without your knowing who will be picked, you are to enter your estimate/guess of what (in inches) you think his/her height will be/is. Once the computer makes its choice, you will have the actual height; enter this in the table. Later complete the table.

Case #	Guess/ Estimate	Actual Height	(Guess - Actual) Signed Error	Squared Error
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
Column Sum/ Totals				

Indicate whether the strategy given here was formulated before, during, or after the presentation of the ten cases above. Describe your strategy and explain/discuss whether it is a good one or not. If you have several strategies, list them; use the back for more space.

Suppose that, for the first case, the student's guess is 60 and that the computer displayed

Name	Height (in inches)
JANICE	62

Then the first row of the table would be:

Case #	Guess/Estimate	Actual Height	Signed Error (Guess - Actual)	Squared Error
1.	60	62	-2	4

For the second case, let's suppose that the student's guess is 67 and that the computer displays

Name	Height (in inches)
KAHI	64

Then, of course, we would have:

Case #	Guess/Estimate	Actual Height	Signed Error (Guess - Actual)	Squared Error
1.	60	62	-2	4
2.	67	64	+3	9

This was continued for the allowed ten cases on the worksheet. Each student then complete his/her table to find his/her sum of squared errors. The class discussion provided a forum for sharing ideas and comparing the results to see who, if anyone, had the smallest sum of squared errors. This went along with identifying and evaluating guessing strategies and further clarification and/or reflection on the concepts of randomness and predictability.

The main objective relative to SSE, however, was to provide students with an objective/mathematical means for assessing closeness for guessing strategies. In addition to the above height-guessing activity which was used to introduce the SSE, a second game-like situation was devised to give students individual, interactive experiences to use and thereby reinforce this quality of SSE.

The SSE-game is interactive in nature and designed for one player. It gives the individual a choice of dealing with whole numbers for 0 to 100 or with the full range of values available in Apple Basic. The user is also given a choice as to the number of values he/she wishes to try to guess, from 1 to 10. The randomly picked undisclosed numbers are selected in order and retain their sequential identity. Thus, the individual is to guess the 1st value, the 2nd value, and so on, and inputs his/her choices accordingly. A correct guess is indicated at the time of input by the bell sound, otherwise, there is no provision in the computer program to retain correct values once identified. The user must, therefore, input the full set of guess values

each pass. Following such input, the computer gives a read-out of the SUM OF SQUARED ERRORS. This is computed via

$$\sum_{i=1}^{n} (G_i - X_i)$$

where n corresponds to the number of values being attempted, X_i to the ith unknown randomly chosen value, and G_i corresponds to the ith value input as a guess for X_i. The computer then sets up for the next round of guesses, unless, of course, all of the values are guessed correctly (SSE=0). In this case, the computer informs the user that he/she is correct and indicates the number of rounds of guesses that were used to solve the problem.

Students were provided with the following lab worksheet to organize their work, to provide feedback to the developer on the operation of the game, to get a record of the strategies that the students used, and to gather information about motivational and attitudinal factors.

Dr. Bernard
3/14/85

Math Eight
SSE Game Score Sheet

Name _____

Date _____

Play one or two games at guessing only one number and with the boundaries set between 0 and 100. Then move on to two, then three, or more numbers, with the boundaries set between 0 and 100. If you REALLY get good at it, you might try letting the boundaries vary. Use the following table to record your SUCCESS!

Game #	# of values to be guessed	Boundaries set, 0 - 100 or vary	Success/ Abandon	Number of Steps USED
		Totals ---		

#1. Describe any special strategies or mathematical principles that you used or discovered in doing the above problems.

#2. What, if anything, did you enjoy, about this activity? What should be kept and what should be changed to make it better?

#3. What kind of special qualities do you now know regarding the Sum of Squared Errors?

#4. Play one of the games with three numbers and try using a single value as your guess each time. What relationship does the best such value have to the three that you are trying to guess? (Use the back if necessary.)

There was a wide range of ability in terms of speed, number of steps, and number of values attempted. Primarily, students played the 0-100 whole number version with from 1 to 3 unknowns to be guessed. Most played at least five games during the 40-minute lab period (with two students sharing and taking turns on a computer.)

Interest ran high and there were no significant negative comments, even from the two or three female students who repeatedly indicated their dislike for working with computers. Many students indicated their enjoyment by requesting that they be given additional opportunities to play the game. This was done.

The basic strategy eventually used by all students was to try a set of values and to alter only one of the guesses while adjusting for the effects on the SSE. For some, this "holdiong all fixed but one" was a significant piece of learning. Some showed more insight than this in recognizing square numbers in the SSE and going up or down an amount equal to the square root of such. Too, some formed an "average squared error" by dividing through by the number of sources of error, usually rounding off, and, in some cases, then taking the square root. This ROOT MEAN SQUARED ERROR (Notice the connection to other areas of statistics, the standard deviation, for example.) served as an amount to try in the going up or down case. One student, subsequently, used a calculator to execute this strategy and became quite skilled at correctly guessing a large number of values in relatively few steps.

The last item of the worksheet reguested that a game be played guessing three values, but that the same input value be input throughout a given input round. It was designed so that, along with the root-mean-squared-error strategy, there would be a natural link to computing standard deviations for a data set. Since the lab time was insufficient to allow any serious work on this item, the item was subsequently handled in a class discussion. Via a teacher-lead experience, students were led to see that the single-shot value which minimizes the sum of squared errors is the arithmetic mean of the set of unknown values. Too, this acts as a balance or pivot point from which one can go up or down to hit target values.

The height-guessing activity and the SSE-Game worked well to help students learn the SSE concept and its properties. They brought out several essential elements of understanding, namely:

1. the sequence of operations used in its computation,

2. that the errors are measured by the differences (subtractions) in the "formula",

3. that the squaring of the errors gives SSE a non-negative quality, with zero as its minimum,

4. that SSE increases rapidly as the errors increase in size, and

5. that there is an associated increase in SSE corresponding to an increase in the number of sources of error.

At the same time, they introduced students to the type of applications that make use of SSE as an objective/mathematical means for evaluating closeness. A discussion of the developmental project work for applications to standaard deviation and best fit modeling follow.

Linking SSE to Standard Deviation

This next activity was developed to bridge the gap between the root mean squared error processes mentioned earlier and standard deviation. Here, the computer was programmed to randomly pick a batch of, say, ten heights (with replacement) that the students were to guess. (Note: Heights were chosen from a pool of data for class members.) The students were provided with appropriately designed worksheets and required to "blindly" enter their guesses before the computer run. From the run, they would get and enter the actual values on the worksheet. Subsequent columns were provided for computing errors/deviations, and the sum of the squared errors/deviations.

Students were surveyed to determine those having the smallest SSE's (notice the link to "best-fit" modeling) and asked to discuss their guessing strategies. The first phase was "open" with students competing against each other.

In the next phase of this same activity, the teacher got into the act with the "novel"/"unusual" strategy of always guessing with the same set number. (Unknown to and to be discovered by the students, the single number used was the arithmetic mean for the population from which samples were being drawn.) Some students were able to occasionally beat (get a lower SSE than) this strategy but came to realize that it was hard to do consistently. They were encouraged to try "one-shot" approaches too and to try to determine if any one single number worked better than others. Many used their own height as a set value. The tallest and shortest heights were also tried as such. In the small amount of time allotted, the students did not automatically propose explicit use of measures of central tendency (mean, median, mode) by name; but some students intuitively opted for values somewhere "near the middle". Thus, for the project, students were prompted to consider the mean, median, and mode; not a very satisfying state of affairs for a teacher/curriculum developer interested in student lead/produced discoveries. Further developmental work might be done in this area. A proper amount of time for student exploration and reflection is needed. Some form of not-too-suggestive, not-too-subtle refresher/re-activating activities for the measures of central tendency and their respective qualities might be found and included with careful timing and sequencing so as to increase the likelihood of students making this discovery in a reasonable amount of time.

Dr. Bernard
2/12/85

Math Eight
"Single-Shot" Height Guessing Strategy
Development

Name _____

Date _____

This is a continuation of an activity you did before. Names of students will be picked at random. You are to guess what you think the student's height will be beforehand. The tables provide for entering your guess, the actual height, and then, for computing signed errors and squared errors, and their respective sums. These may be used to judge the quality of your guessing strategy.

In this phase, you may want to consider that the mean of the group from which the choices are made is 64.74, the median is 65, and the mode is 62. Also, you will be asked to check the possibility of using one of these set values as the one and only guess throughout.

	(Guess - Actual = Error)				(Guess - Actual = Error)			
Case #	Guess/ Estimate	Actual Height	Signed Error	Squared Error	Guess/ Estimate	Actual Height	Signed Error	Squared Error
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
Column Totals								
	<---------"Single-Shots"---------> Guess				<----------Your Choice---------->			

Discuss here and continue on the back, is it reasonable to attempt a stategy of guessing a single value that hopefully will work to give the "BEST" results over the long run? Compare this to other possible strategies. Provide additional information or comments that you feel are important.

In the project setting, SSE was tied to the standard deviation for a data set with the aid of a computer program which produced output "just like" the four-column worksheets that students used to test their guessing strategies and skills "measured by" the sum of squared errors/deviations. The data was incorporated into the program by means of a DATA statement and printed in the first column. The second column contained the "single-shot" guess that was input by the user. The third and fourth columns provided corresponding signed errors/deviations and squared errors/deviations. Column sums were provided and, in addition to what was on student worksheets, the computer produced the mean squared error and the root mean squared error. The mean for the data set was computed in class by the program. Students recorded the results, as in a table:

Input	SSE	Mean Squared Error	Root Mean Squared Deviation
_____	___	_____	_____
_____	___	_____	_____

Through class discussion and several trials the students came to understand the principle that, "the smallest possible root mean squared deviation was produced when deviations were 'measured' from the arithmetic mean for the data set"; too, that, "this was correlated with producing the minimum SSE". It was explained that this was a significant finding in statistics and tied to what mathematicians call the standard deviation. Follow-up lessons explicated the computation procedures (Note: here dividing by n, not n-1, was used. Students were alerted to the possibility of a need for the latter when using a sample statistics to estimate the value for the "larger" population.) and discussed the use of the standard deviation as a unit for measuring above and below the mean for a distribution. The latter was also used in assessing the common vs unusual nature of sample items selected from the distribution, and hypothetical cases as well.

Using SSE to Match A Priori and Empirical Outcomes

By the time students reach intermediate school, either through social interactions or in school, many students seem to have acquired, if not the understanding, then at least the information in one or more of the following "facts":

i. if a fair/balanced coin is tossed, the probability of getting a head is one-half,
ii. if a fair/unloaded die is tossed/thrown, the probability of getting a six (or a one) is one-sixth,
iii. if a single card is chosen/picked/selected at random, from an ordinary deck of fifty-two playing cards, the probability of getting a heart (or diamond, or club, or spade) is one-fourth, or
iv. that they have a 50-50 (fifty-fifty) chance of getting a TRUE-FALSE test item correct just by guessing.

To a lesser extent, they might also "know" that

v. if/when two fair coins are tossed, the probability of both showing heads is one-fourth, and
vi. if/when two fair dice are tossed, the probability of getting a sum of seven is one-sixth.

In some cases these "facts" are so deeply in-grained that it presents a monumental challenge to curriculum writers and teachers to get students to be reflective when such common situations are discussed in order to create an understanding of the all important distinction between a priori analyses/assumptions and empirical/experimentally observed realities. Sometimes there is merit in creating a bit of cognitive dis-equilibrium by such things as having students test their predictions repeatedly for the number of resulting heads when **five** coins are tossed; what about "getting two and one-half heads?!" A set of curriculum materials (see reference for the Intermediate School Mathematics Program), developed and used at the University of Hawaii Laboratory School, seems to have provided a good basis for addressing these issues. At one point, the materials provide data collection activities, with records first made by students working in pairs followed by pooling the resulting tallies for the entire class, for the eight experiments;

i. rolling an ordinary, fair die 100 times,
ii. tossing a fair coin 50 times,
iii. spinning a two-equal-sections spinner 50 times,
iv. tossing a thumbtack 50 times,
v. picking (with replacement) one bead from a grab-bag containing 2 red,
3 yellow, and 5 blue beads, 50 times,
vi. tossing two dice (observing sums) 100 times,
vii. tossing two coins (HH, TT, MIXED) 50 times, and
viii. choosing a card (with replacement, and shuffling), recording its suit 50 times.

For project students, the experimentally found frequencies were then used with the formula:

$$\text{Relative frequency for a selected outcome} = \frac{\#\text{ of ocurrences of the outcome}}{\text{Total \# of trials in the experiment}}$$

Both formats, decimal (to thousandths) and percents (to tenths) were used to express and discuss the results.

Many authors and teachers slight over or are grossly misleading about the true connection between theoretical/a priori probabilities and relative frequencies. It is not uncommon to find statements such as, "Well, if you make n, the number of trials, larger and larger, you'll get it." For example, the relative frequency for heads for 30 tosses of a particular coin for a particular experiment might happen to be 0.468. The misleading claim might then be that, if you increased the number of tosses to 100, to 1,000,000, then you'd get 1/2. Some might temper this by saying you'd get closer and closer to 1/2, or some other variation; few of which, if any, are totally adequate. Admittedly this is a cumbersome situation to tackle at an elementary level. The following, however, was presented to/used with the project group in an attempt to give students an "honest" feeling for the situation.

Project students were provided with a specially designed computer program to assist them in finding the "best fitting" set of rational numbers/common fractions corresponding to the set of decimal relative frequencies found in the experiments. This is illustrated here for the experiment of simultaneously tossing four coins all at once and recording the number of heads. Possible outcomes for the exact number of heads are 0, 1, 2, 3, and 4. Suppose the corresponding relative frequencies from a

large number of trials are respectively 0.058, 0.199, 0.481, 0.195, and 0.067. The computer program requested the number of outcomes, then a name or description and corresponding decimal relative frequency for each. Once this information was entered, the matching/modeling process began.

The types of questions that the program is designed to answer are:

i. If halves were used to model the experiment, which halves (0/2, 1/2, or 2/2) would correspond (come closest) to the given relative frequencies? (Similarly for thirds (0/3, 1/3, 2/3, or 3/3), fourths (0/4, 1/4, 2/4, 3/4, or 4/4), and so on.)
ii. What error and how much squared error results from using the "best fitting model fraction" with a set denominator instead of its corresponding relative frequency?
iii. What is the total sum of squared error resulting from using the entire
set of model fractions with a selected, common denominator in comparison to the empirical results (relative frequencies)?, and
iv. What is the sum of the set of model fractions for a selected denominator?

The following table shows selected results for the 4-coin tossing experiment.

	<——Exact Number of Heads Showing——>						
Outcome	0	1	2	3	4		
Decimal Rel. Freq.	0.058	0.199	0.481	0.195	0.067	Sum of Model Fractions	Sum of Squared Errors
Test/Model Denominator	<——Best Matching Fractions——>						
2	0/2	0/2	1/2	1/2	0/2	1/2	0.086
3	0/3	1/3	1/3	1/3	0/3	3/3	0.067
4	0/4	1/4	2/4	1/4	0/4	4/4	0.014
5	0/5	1/5	2/5	1/5	0/5	4/5	0.014
15	1/15	3/15	7/15	3/15	1/15	15/15	0.000
16	1/16	3/16	8/16	3/16	1/16	16/16	1E-03
17	1/17	3/17	8/17	3/17	1/17	16/17	0.001
1000	58/1000	199/1000	481/1000	195/1000	67/1000	1000/1000	0.000

For the first few experiments, worksheet tables like the above, included some of the denominators and some of the other information, but not all. Project students used the computer to fill in the missing information. Also blank rows were supplied so that students could try test denominators of their own choice and order of input; freedoms which they had then entirely for the later experiments. Thus, after they would put in a test denominator, they would record the desired information received from the computer in one of the rows of the table, while the computer was then made ready for another test denominator.

Worksheet inquiry/discussion questions suggested looking for patterns, making comparisons, and making use of knowledge of probabilities. For example, the students had learned that the sum of the probabilities across an experiment for all outcomes must have a total of one, no more, no less. A particular discussion question might allow for the use of this principle as in the case of asking, "why the set of 17ths does not make a good model for this experiment." The idea that the smaller SSE provides a better fit might be brought out in response to, "why fourths provide a

better model than thirds." Students soon discovered, however, that larger denominators generally (constituting finer and finer measures) produced smaller SSE's, and often with a model sum of one. This allowed a search for other criteria/reasons for choosing a "single best model". Here, combinatoric analyses for producing sample spaces of equally likely possible outcomes entered the picture. Students had had the experience of listing all of the 16 possibilities; TTTT, TTTH,..., HHH. Thus, in the total, sixteenths seemed to provide the best model for this experiment because:

1. The sum of the model fractions is 16/16 = 1
2. The SSE is small, nearly minimal
3. The experiment can be analyzed in terms of 16 equally likely outcomes. (Symmetry could also be discussed here.)

A comment is in order regarding the theoretical values for this experiment. The probability of getting exactly r heads for a single simultaneaous toss of n fair (probability of heads for each individual coin being 1/2) coins is given by ${}_nC_r(1/2)^n$ where ${}_nC_r$ is the binomial coefficient for the combination of n things chosen r at a time. With n=4, r=0,1,2,3, and 4, we have, respectively, 1/16, 4/16, 6/16, 4/16, and 1/16. The work here is seen as a bridge toward the more sophisticated binomial distribution model (schema) which is often tested for its fit to empirical data in more advanced work using the Chi-square statistic. The approach that was taken during the project seems to have a bit of a philosophical flavor to it; the reader might try giving justification/an explanation of why the 1/16, 4/16, 6/16, 4/16, 1/16 set is a better model (or not) than 1/16, 3/16, 8/16, 3/16, 1/16. Thus, the approach goes beyond mere facts, to give students some feeling about establishing scientific "truths" and the limitations on our "knowing" reality.

SSE Applied to Linear Regression

Project work in this area is less complete than for others. Most of the development was centered around in-class teacher-lead computer demonstrations and discussions coupled with worksheets and homework assignments; the design and writing of self-sufficient computer programs for interactive student laboratory experiences was not finished within the time limits set for the project.

The lead-in to this topic was provided by the Intermediate School Mathematics Program materials developed by the University of Hawaii Curriculum Research and Development Group (see reference, 1981). The materials included a section on correlation which included exercises for plotting data points for such comparisons as height versus shoe size, and temperature versus (cricket) chirps on two-dimensional grids (ie., locating the points in a coordinate plane). The results were discussed more-or-less intuitively in terms of whether the "data ball" showed a linear pattern or not, if the relationship was strong or weak, and if it was positive or negative.

These materials also included exercises on prediction equations. For example, the equation W = 5.5H - 220 was said to be able to predict a person's weight, W, (in pounds) from his/her height, H, (in inches). This is what was actually used as the link to SSE. Students realized that the prediction equation produced errors, working "pretty well" in some cases but being "way off" in others. Their feelings were made more explicit by means of exercises using the prediction equations to fill in tables like the following which culminated in finding SSE.

Student ID #	Height (inches)	Predicted Weight (pounds)	Actual Weight (pounds)	Signed Error	Squared Error
1	63	126.5	115.5	_____	121.00
2	65	137.5	130	+7.5	56.25
3	70	_____	170	_____	_____
				Sum of Squared Errors	

Later, students were made aware of the fact that these were linear prediction equations, that their graphs were straight lines, and that among the multitude of possible "lines," the "desired" one was the one which minimized the sum of squared errors for a given set of data.

A computer program which is capable of plotting the data points and superimposing the graph for the "best fitting" line (in the sense of least squares) was used in combined classroom demonstration-discussions to find such lines; including their equations, the corresponding SSE, and R (the coefficient of linear regression). Small data sets of from three to five points, picked, either by the teacher or the students, were used to demonstrate the various qualities. A second program which allowed one to move the line up or down by changing the y-intercept, or by twisting the line making it steeper or not, was used to "solidify" the fact that SSE was minimal for the regression line and larger for any other line. During such discussions it was repeatedly emphasized that the errors in question corresponded to the vertical distance for the difference between the ordinate for the point on the line and the ordinate for a selected data point.

Reference

Intermediate School Mathematics Project, Director: Nancy C. Whitman, Ph.D. Intermediate School Mathematics Program: Book IV. Honolulu, Hawaii: University of Hawaii College of Education, 1981.

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